Semester Examination 2022 Code: MATH/202/22

M.Sc. II Semester

Subject: Mathematics

Paper II

Real Analysis II

Time: 3 Hours

Max. Marks: 80

Min. Marks: 16

Note: The question paper consists of three sections A, B and C. All questions are compulsory.

Section A - Attempt all multi choice questions.

Section B - Attempt one question from each unit.

Section A - Attempt one question from each unit.

SECTION A

$2 \times 8 = 16$

1. Let f be continuous and α monotonically increasing on [a, b], then:

(a) $f \notin R(\alpha)$ on [a, b]

(b) $f \in R(\alpha)$ on [a, b]

(c) $\int_a^b f\, d\alpha = 0$

(d) None of these

2. If P^* is a refinement of P, then:

(a) $L(P, f, \alpha) \leq L(P^*, f, \alpha)$

(b) $U(P^*, f, \alpha) > U(P, f, \alpha)$

(c) $L(P, f, \alpha) > L(P^*, f, \alpha)$

(d) None of these

3. Let A and B be two sets such that $A \subseteq B$, then:

(a) $m^*(A) \neq m^*(B)$

(b) $m^*(A) \leq m^*(B)$

(c) $m^*(A) \geq m^*(B)$

(d) None of these

4. If A and B are disjoint measurable subsets of E, then:

(a) $\int_{A \cup B} f = \int_A f + \int_B f$

(b) $\int_{A \cup B} f \leq \int_A f + \int_B f$

(c) $\int_{A\cup B} f \geq \int_A f + \int_B f$

(d) $\int_{A\cup B} f > \int_A f + \int_B f$

5. Cantor set has measures

(a) 1/3

(b) 1

(c) 0

(d) 2/3

6. Singleton set $\{x\}$ has its measures:

(a) 0
(b) 1
(c) $\frac{1}{2}$
(d) 2

7. An extended real valued function f: E → R defined on Measurable set E. Let E = R, then the set E (f > a) is:

(a) Open set
(b) Closed set
(c) Both (a) and (b)
(d) None of the above

8. If $f(x) \in L^p$ and $g(x) \in L^p$, then

(a) $f(x) - g(x) \in L^p$

(b) $f(x) + g(x) \in L^p$
(c) Both (a) and (b)
(d) None of the above

SECTION B 4 x 6 = 24

1. Define Riemann – Stieltjes integral of a real bounded function defined on [a, b]. Evaluate: $\int_0^3 x\,d([x]-x)$

OR

Let f be continuous function and α monotonically increasing on [a, b] then prove that $f \in R(\alpha)$ on [a, b].

2. Define Borel measurable set. Prove that the symmetric difference of two measurable sets is measurable.

OR

Prove that the outer measure of an interval is its length.

3. Define Caratheodory outer measure. If $A \in \alpha$ (algebra of sets) then prove that $\mu^*(A) = \mu(A)$.

OR

State and prove Fatau's lemma

4 Find the four derivatives for the function $f : R \rightarrow R$ define by $f(x) = |x|$ for all $x \in R$.

OR

If $f \in L^2\,[0,1]$, then show that: $\left|\int_0^1 f(x)dx\right| \leq \left[\int_0^1 |f(x)|^2 dx\right]^{1/2}$

SECTION C 4 x 10 = 40

1. If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$, then prove that $f \in R(\alpha_1 + \alpha_2)$ and

$$\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$$

OR

Let γ be a continuous differentiable curve on [a, b], then prove that γ rectifiable and $\Lambda_y(a, b) = \int_a^b |\gamma'(t)| dt$.

2. Prove that the interval (a, ∞) is measurable.

OR

Define measureable set. Prove that the union of a finite number of measureable set is measureable.

3. State and prove Lebesgue-Monotone convergence theorem.

OR

Let $E_1, E_2, \ldots\ldots\ldots\ldots, E_n$ a finite sequence of disjoint measurable set, then prove that:

$\mu^*(A \cap [\cup_{i=1}^n E_1]) = \sum_{i=1}^n \mu^*(A \cap E_i)$ holds for every subset A of X

4. State and prove Vitali's covering Lemma.

OR

State and prove Riesz-Fischer theorem.

$\mu^*(A \cap (\bigcup_{i=1}^n E_i)) = \sum_{i=1}^n \mu^*(A \cap E_i)$